3 $\begin{vmatrix} 1 & 2 & x \\ 2 & 1 & y \\ 0 & 1 & 2 \end{vmatrix} = 3, \Rightarrow \begin{vmatrix} 1 & 2 & x+1 \\ 2 & 1 & y \\ 0 & 1 & 2 \end{vmatrix} =$ 1 5 (填入整數或分數)

$$0\begin{vmatrix} 2 & x \\ 1 & y \end{vmatrix} - 1\begin{vmatrix} 1 & x \\ 2 & y \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 3$$

$$\Rightarrow 0 - 1(y-2x) + 2(1-4) = 3$$

$$\Rightarrow -y+2x+(-6) = 3$$

$$\Rightarrow -y+2x$$

$$\begin{vmatrix} 1 & 2 & x+1 \\ 2 & 1 & y \\ 0 & 1 & 2 \end{vmatrix} = 0\begin{vmatrix} 2 & x+1 \\ 1 & y \end{vmatrix} - 1\begin{vmatrix} 1 & x+1 \\ 2 & y \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= 0 - 1(y-(2x+2)) + 2(-3)$$

$$= -1(y-2x-2) - 6$$

$$= -y+2x+2-6$$

$$= -y+2x-4$$

$-6-(-4) = -2$

少扣2，所以+2

$\therefore 3+2=5$

4 若$A=\begin{bmatrix}2 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -2\end{bmatrix}$, 則$[A^3]_{33}=$ 64 -8 ，$[A^{-3}]_{33}=$ 1/64 -1/8 (填入整數或分數)

我算成行列式值...

(1)

$(-2)^3=-8$

(2) $-\frac{1}{8}$

5 $\begin{vmatrix} x & x+1 & x+2 \\ x+2 & x & x+1 \\ x+1 & x+2 & x \end{vmatrix} = 0,\ \Rightarrow x =$ 0 -1 (填入整數或分數)

$$x\begin{vmatrix} x & x+1 \\ x+2 & x \end{vmatrix} - (x+1)\begin{vmatrix} x+2 & x+1 \\ x+1 & x \end{vmatrix} + (x+2)\begin{vmatrix} x+2 & x \\ x+1 & x+2 \end{vmatrix} = 0$$

$$\Rightarrow\ x\left(x^2-(x+1)(x+2)\right) = x\left(x^2-(x^2+2x+x+2)\right) = x(-2x-x-2) = -3x^2-2x$$

$$-(x+1)\left((x^2+2x)-(x^2+2x+1)\right) = -(x+1)(-1) = x+1$$

$$(x+2)\left((x^2+4x+4)-(x^2+x)\right) = (x+2)(3x+4) = 3x^2+4x+6x+8 = 3x^2+10x+8$$

$$\therefore\ \cancel{-3x^2}-2x+x+1+\cancel{3x^2}+10x+8$$

$$= 9x+9$$

$$= 0$$

$$\therefore\ x=-1$$

6

線性系統 $\begin{cases} x+2y-z=1 \\ 2x+3y-4z=-3 \\ 3x+6y-3z=4 \end{cases}$ 的解為

我當時沒發現第一列跟第三列成比例
於是我用高斯消去法去做

Sol:

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 3 & -4 & -3 \\ 3 & 6 & -3 & 4 \end{array}\right] \begin{array}{l} \times(-2) \\ \\ \times(-3) \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -1 & -2 & -5 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

我沒意識到 $0 \neq 1$，反而想成無限多解
不知道我再搞什麼

$\therefore 0 \neq 1$，無解

7

Q1. $A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 0 \\ 4 & 2 & 18 \end{bmatrix} = LU, L = [l_{ij}]$為下三角，$U = [u_{ij}]$為上三角，則$l_{32} =$ 4/3 4 (填入整數或分數)

Q2. 將$A = LU$分解的計算過程寫在答案紙上，本小題編號為"4"，分數另計。

Sol:

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 0 \\ 4 & 2 & 18 \end{bmatrix} \begin{matrix} \leftarrow \times\frac{1}{2} \\ \\ \\ \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ -4 & 5 & 0 \\ 4 & 2 & 18 \end{bmatrix} \begin{matrix} \times 4 \\ \leftarrow \\ \\ \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 3 & 6 \\ 4 & 2 & 18 \end{bmatrix} \begin{matrix} \times(-4) \\ \\ \leftarrow \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 3 & 6 \\ 0 & 4 & 12 \end{bmatrix} \begin{matrix} \\ \leftarrow \times\frac{1}{3} \\ \\ \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 2 \\ 0 & 4 & 12 \end{bmatrix} \begin{matrix} \\ \times(-4) \\ \leftarrow \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = U$$

$$E_1 = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

$$E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{bmatrix}$$

$$\therefore E_5E_4E_3E_2E_1A = U \Rightarrow L = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}E_5^{-1}$$

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ -4 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ -4 & 3 & 0 \\ 4 & 4 & 1 \end{bmatrix}$$

$$\therefore \; [L]_{32} = 4$$

9

Q1 利用克拉瑪法則求解系統 (本小題的解須寫在答案紙上，試題編號為"2"，分數另計)

$$\begin{cases} 2x - 3y = -2 \\ x + 2z = 5 \\ x + y + z = 0 \end{cases}$$

Q2 上述的解中 x = -11/7 -19/7 (填入整數或分數)

令 $A = \begin{bmatrix} 2 & -3 & 0 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$

$$\Delta = \begin{bmatrix} 2 & -3 & 0 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= 2\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} - (-3)\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$$

$$= 2 \cdot (-2) - (-3)(-1)$$

$$= -4 - 3 = -7$$

$$x = \frac{1}{-7}\begin{bmatrix} -2 & -3 & 0 \\ 5 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix} = -\frac{1}{7}\left(-1\begin{vmatrix} -2 & 0 \\ 5 & 2 \end{vmatrix} + 1\begin{vmatrix} -2 & -3 \\ 5 & 0 \end{vmatrix}\right)$$

$$= -\frac{1}{7}(4+15) = \frac{-19}{7}$$

$$y = \frac{1}{-7}\begin{bmatrix} 2 & -2 & 0 \\ 1 & 5 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{-7}\left(1\begin{vmatrix} -2 & 0 \\ 5 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & -2 \\ 1 & 5 \end{vmatrix}\right)$$

$$= \frac{1}{-7}(-4+12)$$

$$= \frac{8}{-7}$$

$$z = \frac{1}{-7}\begin{bmatrix} 2 & -3 & -2 \\ 1 & 0 & 5 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \frac{1}{-7}\left(-1\begin{vmatrix} -3 & -2 \\ 1 & 0 \end{vmatrix} - 5\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}\right)$$

$$= \frac{1}{-7}\left(-(2) - 5(2+3)\right)$$

$$= \frac{1}{-7}(-2-25)$$

$$= \frac{1}{-7}(-27)$$

$$= \frac{27}{7}$$