$\det(kA) = k^n \det(A)$

kA中共有n個列有共同倍數k

$\det(A+B) \neq \det(A) + \det(B)$

ex:

$A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$, $\det(A) = 5-4 = 1$

$B = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$, $\det(B) = 9-1 = 8$

$A+B = \begin{bmatrix} 4 & 3 \\ 3 & 8 \end{bmatrix}$, $\det(A+B) = 32-9 = 23$

$\therefore \det(A+B) \neq \det(A) + \det(B)$

令 A. B, C 為只有第r列不同的 $n \times n$ 矩陣,

C的第r列為A、B第r列的合

$\Rightarrow \det(A) + \det(B) = \det(C)$

$$\det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \det\begin{bmatrix} a_{11} & a_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \det\begin{bmatrix} a_{11} & a_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}$$

矩陣乘積之行列式值

if A、B為大小相同的方陣

$$\det(AB) = \det(A)\det(B)$$

ex: $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, $\det(A) = 1$

$B = \begin{bmatrix} -1 & 3 \\ 5 & 8 \end{bmatrix}$, $\det(B) = -23$

$AB = \begin{bmatrix} 2 & 17 \\ 3 & 14 \end{bmatrix}$, $\det(AB) = -23$

$$\therefore \det(AB) = \det(A)\det(B)$$

若為可逆矩陣 $\Rightarrow \det(A^{-1}) = \dfrac{1}{\det(A)}$

Because $A^{-1}A = I$, $\det(A^{-1}A) = \det(I) = 1$

$\therefore \det(A)\det(A^{-1}) = 1$

$\Rightarrow \det(A^{-1}) = \dfrac{1}{\det(A)}$

伴隨矩陣 adj

def: 餘因子矩陣的轉置矩陣

ex: $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$

$$\det(A) = 2\begin{vmatrix} 2 & -1 \\ 6 & 3 \end{vmatrix} - (-4)\begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix}$$

$$= 2\cdot(6+6) - (-4)\cdot(9+1)$$

$$= 24 + 40 = 64$$

$$A^{-1} = \frac{1}{64}\,\operatorname{adj}(A)$$

$$\begin{array}{ccccc} \multicolumn{3}{c}{\left[\begin{array}{ccc} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{array}\right]} & \begin{array}{c} 3 \\ 1 \\ 2 \end{array} & \begin{array}{c} 2 \\ 6 \\ -4 \end{array} \\ 3 & 2 & -1 & 3 & 2 \\ 1 & 6 & 3 & 1 & 6 \end{array}$$

$$\Rightarrow \operatorname{adj}(A) = \begin{bmatrix} 12 & 4 & 12 \\ 6 & -2 & -10 \\ -16 & 16 & 16 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{64}\begin{bmatrix} 12 & 4 & 12 \\ 6 & -2 & -10 \\ -16 & 16 & 16 \end{bmatrix}$$

克拉馬法則解線性系統

$$
\begin{aligned}
x_1 + \quad + 2x_3 &= 6 \\
-3x_1 + 4x_2 + 6x_3 &= 30 \\
-x_1 - 2x_2 + 3x_3 &= 8
\end{aligned}
$$

$$\Delta = \begin{vmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{vmatrix} = 1(12+12) + 2(6+4)$$

$$= 24 + 20 = 44$$

$$x_1 = \frac{1}{44} \begin{vmatrix} 6 & 0 & 2 \\ 30 & 4 & 6 \\ 8 & -2 & 3 \end{vmatrix}$$

$$= \frac{1}{44}\left(6(12+12) + 2(-60-32)\right)$$

$$= \frac{1}{44}(144 - 184)$$

$$= \frac{-40}{44}$$

$$X_2 = \frac{1}{44}\begin{vmatrix} 1 & 6 & 2 \\ -3 & 30 & 6 \\ -1 & 8 & 3 \end{vmatrix}$$

$$= \frac{1}{44}\left(1(90-48)-6(-9+6)+2(-24+30)\right)$$

$$= \frac{1}{44}(42+18+12)$$

$$= \frac{72}{44}$$

$$X_3 = \frac{1}{44}\begin{vmatrix} 1 & 0 & 6 \\ -3 & 4 & 30 \\ -1 & -2 & 8 \end{vmatrix}$$

$$= \frac{1}{44}\left(1(32+60)+6(6+4)\right)$$

$$= \frac{1}{44}(92+60)$$

$$= \frac{152}{44}$$