$A \rightarrow$ 方陣 $\rightarrow$ 任一列 or 行為 0 $\Rightarrow \det(A) = 0$

$A \rightarrow$ 方陣 $\rightarrow \det(A) = \det(A^T)$

A 為 $n \times n$ 矩陣

若 B 矩陣為 A 矩陣之一列 or 一行乘 k 倍

$\Rightarrow \det(B) = k \det(A)$

若 B 矩陣為 A 矩陣之兩列 or 行交換結果

$\Rightarrow \det(B) = -\det(A)$

若 B 矩陣為 A 矩陣之一列乘上某倍加到某行 or 列

$\Rightarrow \det(A) = \det(B)$

兩列成比例 $\Rightarrow$ determinant $= 0$

以列運算求det

ex:

$$A = \begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}$$

Sol:

$$\det(A) = \begin{vmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{vmatrix}$$

$$= - \begin{vmatrix} 3 & -6 & 9 \\ 0 & 1 & 5 \\ 2 & 6 & 1 \end{vmatrix}$$

$$= -3 \begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 2 & 6 & 1 \end{vmatrix} \times(-2)$$

$$= -3 \begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 10 & -5 \end{vmatrix} \times(-10)$$

$$= -3 \begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & -55 \end{vmatrix} = -3 \times -55 = 165$$

使用列運算和餘因子展開計算det

$$A = \begin{array}{c l} \begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 3 & 7 & 5 & 3 \end{bmatrix} & \begin{array}{l} \times(-3) \\ \\ \times(-2) \\ \times(-3) \end{array} \end{array}$$

Sol:

$$\det(A) = \begin{vmatrix} 0 & -1 & 1 & 3 \\ 1 & 2 & -1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 1 & 8 & 0 \end{vmatrix}$$

$$= - \begin{array}{c l} \begin{vmatrix} -1 & 1 & 3 \\ 0 & 3 & 3 \\ 1 & 8 & 0 \end{vmatrix} & \times 1 \end{array}$$

$$= - \begin{vmatrix} -1 & 1 & 3 \\ 0 & 3 & 3 \\ 0 & 9 & 3 \end{vmatrix}$$

$$= -(-1)(9-27)$$

$$= -18$$